

Brief communication

# Assessment of a statistical model for the transport of discrete particles in a turbulent channel flow

B. Arcen<sup>a</sup>, A. Tanière<sup>a,\*</sup>, L.I. Zaichik<sup>b</sup>

<sup>a</sup> LEMTA, Nancy-University, CNRS, ESSTIN, 2 Rue Jean Lamour, F-54529 Vandoeuvre-lès-Nancy, France

<sup>b</sup> Nuclear Safety Institute of the Russian Academy of Sciences, B. Tulsukaya, 115191 Moscow, Russia

Received 28 May 2007; received in revised form 19 October 2007

*Keywords:* Gas–solid flows; DNS; PDF model; Inhomogeneous turbulence

## 1. Introduction

In the frame of the PDF modelling approach, we propose in the present paper to study the performance of relations for the mean turbulent particle and fluid–particle properties derived by Zaichik et al. (2004) from a statistical model for the transport of particles in quasi-homogeneous turbulent flows. This model, called Anisotropic TimeScale model (ATS), has the advantage over the majority of continuum two-fluid models of taking the directional dependence of the timescales of the turbulence into account. Assuming these timescales identical to those obtained along fluid-element trajectories, it was demonstrated by Zaichik et al. (2004) that this model reproduces correctly the crucial trends of the particle statistics in a linear shear turbulent flow.

Nevertheless, in the model proposed by Zaichik et al. (2004), the correlations of the fluid fluctuating velocity viewed by a particle were assumed to be identical to those obtained along fluid-element trajectories due to the difficulty of predicting the inertia and crossing trajectory effects on these correlations (Wang and Stock, 1993). Therefore, the statistical bias that exist between the turbulent properties of the fluid viewed by a particle and those of the fluid were neglected. In the present study, we propose to evaluate the performance of two versions of the ATS model. In the first one, the inertia and crossing trajectory effects on the turbulent properties of the fluid viewed by a particle

are taken into account, whereas the second version is identical to the one proposed by Zaichik et al. (2004). Knowing the importance of introducing the turbulent properties of the fluid seen by a particle in the ATS model, we will be able to conclude on the possibility to overcome the problem of knowing a priori these properties.

In order to evaluate the performance of the two versions of the ATS model in non-homogeneous turbulent flows, results obtained from a direct numerical simulation of a gas-particle channel flow are compared to those predicted by these models. This investigation is conducted at moderate Reynolds number without taking the gravity force into account, and for three sets of particles characterized by different relaxation times. It has to be noted that in this flow configuration, the crossing trajectory effect can appear due to the existence of a nonzero mean relative velocity between the fluid and the particles (see Ferry et al., 2003). Nevertheless, this effect cannot be considered as significant due to the low magnitude of the mean relative velocity (Arcen et al., 2004). Consequently, the statistical bias between the turbulent properties of the fluid viewed by a particle and those of the fluid is mainly induced by the inertia effect.

## 2. PDF model description

In this section, we present the modified version of the ATS model in which the inertia and crossing trajectory effects on the turbulent properties of the fluid viewed by a particle are taken into account. The theoretical ground of the model being considered is a kinetic equation for

\* Corresponding author. Tel.: +33 383 685 083.

E-mail address: [anne.taniere@esstin.uhp-nancy.fr](mailto:anne.taniere@esstin.uhp-nancy.fr) (A. Tanière).

the PDF of the particle velocity. This kinetic equation describes the interaction of particles with fluid turbulent eddies. The operator providing the particle–turbulence interaction in the kinetic equation was derived with modelling the fluid turbulence by a Gaussian random process and using the functional formalism (Zaichik, 1999; Zaichik et al., 2004). To focus upon the transport of particles, we consider the so-called one-way coupling between particles and turbulent eddies. By this is meant that the particle volume and mass fractions are small enough such that particle–particle interactions and turbulence modulation by particles can be neglected. Moreover, we assume that the particle density is much more than that of the carrier fluid (in this case, the drag force acting on a particle is only of importance), and the particle size is small as compared to the Kolmogorov lengthscale. In such a statement, the kinetic equation for the PDF of the particle velocity distribution,  $P(\mathbf{x}, \mathbf{v}, t)$ , has the following form (Zaichik et al., 2004):

$$\begin{aligned} \frac{\partial P}{\partial t} + v_i \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial v_i} \left[ \left( \frac{\langle u_i \rangle - v_i}{\tau_p} + F_i \right) P \right] \\ = \lambda_{ij} \frac{\partial^2 P}{\partial v_i \partial v_j} + \mu_{ij} \frac{\partial^2 P}{\partial x_j \partial v_i}, \end{aligned} \quad (1)$$

where  $F_i$  denotes the external force acting on a particle per unit mass, and  $\tau_p$  is the Stokes particle response time which can be expressed in terms of the particle diameter ( $d_p$ ), the particle and fluid densities ( $\rho_p$  and  $\rho_f$ ), and the kinematic viscosity ( $\nu$ ) as  $\tau_p = (\rho_p d_p^2) / (18 \rho_f \nu)$ . The coefficients  $\lambda_{ij}$  and  $\mu_{ij}$  are given by:

$$\begin{aligned} \lambda_{ij} = \langle \tilde{u}'_i \tilde{u}'_k \rangle \left( \frac{f_{kj}}{\tau_p} + l_{kn} \frac{\partial \langle \tilde{u}_j \rangle}{\partial x_n} + \tau_p m_{kl} \frac{\partial \langle \tilde{u}_n \rangle}{\partial x_l} \frac{\partial \langle \tilde{u}_j \rangle}{\partial x_n} \right), \\ \mu_{ij} = \langle \tilde{u}'_i \tilde{u}'_k \rangle \left( g_{kj} + \tau_p h_{kn} \frac{\partial \langle \tilde{u}_j \rangle}{\partial x_n} \right), \end{aligned} \quad (2)$$

where  $\langle \tilde{u}_i \rangle$  and  $\langle \tilde{u}'_i \tilde{u}'_k \rangle$  are the mean velocity and Reynolds stresses of the fluid seen by particles. To introduce the statistical bias between the turbulent properties of the fluid viewed by a particle and those of the fluid, the mean velocity and Reynolds stresses of the carrier phase have been substituted by the Reynolds stresses of the fluid seen in the original definition of the response coefficients proposed by Zaichik et al. (2004). In Eq. (2), the coefficients  $f_{ij}$ ,  $l_{ij}$ ,  $m_{ij}$ ,  $g_{ij}$  and  $h_{ij}$  measure the response of particles to velocity fluctuations of the fluid and are related to the time-scales tensor of the fluid velocity viewed by a particle.

The terms on the left- and right-hand sides of the kinetic equation (Eq. (1)) describe, respectively, the convective transport of the PDF in phase space and the diffusion transfer caused by the particle–turbulence interaction. Modelling the fluid turbulence by means of a Gaussian process enables the particle–turbulence interaction to be expressed explicitly in the form of the second-order differential operator. This diffusion operator takes proper account of the effect of timescale anisotropy through the response coefficients. In the case of negligible timescale

anisotropy, the response coefficients are scalars rather than tensors. The kinetic equation completely controls the velocity statistics of the particulate phase. However, for most practical purposes, the kinetic level of modelling is not only computationally too expensive, but is also unnecessary because macroscopic properties are usually all that are needed. Another computationally less expensive way is to solve the conservation equations for several first moments of the PDF. The kinetic equation (Eqs. (1) and (2)) generates the following set of governing continuum equations describing the conservation of mass, momentum, and particulate stresses as the appropriate statistical moments of the particle velocity PDF:

$$\frac{\partial \Phi}{\partial t} + \frac{\partial \Phi \langle v_{p,k} \rangle}{\partial x_k} = 0, \quad (3)$$

$$\begin{aligned} \frac{\partial \langle v_{p,i} \rangle}{\partial t} + \langle v_{p,k} \rangle \frac{\partial \langle v_{p,i} \rangle}{\partial x_k} = - \frac{\partial \langle v'_{p,i} v'_{p,k} \rangle}{\partial x_k} + \frac{\langle \tilde{u}_i \rangle - \langle v_{p,i} \rangle}{\tau_p} \\ + F_i - \frac{D_{p,ik}}{\tau_p} \frac{\partial \ln \Phi}{\partial x_k}, \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial \langle v'_{p,i} v'_{p,j} \rangle}{\partial t} + \langle v_{p,k} \rangle \frac{\partial \langle v'_{p,i} v'_{p,j} \rangle}{\partial x_k} + \frac{1}{\Phi} \frac{\partial \Phi \langle v'_{p,i} v'_{p,j} v'_{p,k} \rangle}{\partial x_k} \\ = - \left( \langle v'_{p,i} v'_{p,k} \rangle + \mu_{ik} \right) \frac{\partial \langle v_{p,j} \rangle}{\partial x_k} - \left( \langle v'_{p,j} v'_{p,k} \rangle + \mu_{jk} \right) \frac{\partial \langle v_{p,i} \rangle}{\partial x_k} \\ + \lambda_{ij} + \lambda_{ji} - 2 \frac{\langle v'_{p,i} v'_{p,j} \rangle}{\tau_p}. \end{aligned} \quad (5)$$

Here  $\Phi$ ,  $\langle v_{p,i} \rangle$ , and  $\langle v'_{p,i} v'_{p,j} \rangle$  are the particle volume fraction, the mean particle velocity, and the particle kinetic stresses, respectively. In Eq. (4),  $D_{p,ik} = \tau_p \left( \langle v'_{p,i} v'_{p,k} \rangle + \mu_{ik} \right)$  corresponds to the particle diffusivity tensor.

On the basis of this PDF approach, Zaichik et al. (2004) also derived the following expressions for the fluid–particle fluctuating velocity covariances,  $\langle \tilde{u}'_i v'_{p,j} \rangle$ :

$$\langle \tilde{u}'_i v'_{p,j} \rangle = \tau_p \left( \lambda_{ij} - \mu_{ik} \frac{\partial \langle v_{p,j} \rangle}{\partial x_k} \right). \quad (6)$$

It should be noted that the influence of the mean fluid and particle velocities is taken into account in these expressions.

In what follows, we now consider the developed flow in a plane channel. All the characteristics of the carrier and dispersed phases are assumed to be self-similar with respect to the longitudinal coordinate,  $x_1$ , and to depend only on the coordinate normal to the wall,  $x_2$ . The channel walls are impermeable and particle deposition is absent. Accordingly, the normal components of the averaged velocity of both phases vanish ( $\langle u_2 \rangle = \langle v_{p,2} \rangle = 0$ ). Moreover, in this study we assume that the particle velocity fluctuations are locally homogeneous and the mean particle velocity is identical to the mean velocity of the fluid seen by particles. These assumptions are valid only for not-too-high-inertia particles when both the contribution of the transport effect

to the particle velocity fluctuations and the difference between the mean fluid and particle velocities are not of very importance.

The sets of equations obtained for the particle kinetic stresses and the fluid–particle covariances are slightly different from those proposed by Zaichik et al. (2004) due to the substitution of the turbulent properties of the carrier phase by those of the fluid viewed by a particle in the specification of the kinetic equation coefficients  $\lambda_{ij}$  and  $\mu_{ij}$ . The model proposed in this study for the particle kinetic stresses and the fluid–particle covariances will be referred to hereafter as modified ATS model, whereas the one proposed by Zaichik et al. (2004), which are function of the fluid Eulerian and Lagrangian statistics, will be referred to as original ATS model.

### 3. Gas–solid channel flow DNS

The DNS solver, second order accurate in space and in time (Orlandi, 2000), performs the simulation of a turbulent channel flow at  $Re_b = 2800$  (based on channel half-width  $\delta$  and bulk velocity  $U_b$ ) corresponding to the Reynolds number based on the wall-shear velocity equal to 184. The domain size in the streamwise, wall-normal, and spanwise direction is  $2.5\pi\delta \times 2\delta \times 1.5\pi\delta$  and the corresponding grid  $192 \times 129 \times 160$ , respectively. The assumptions and the numerical methods used to simulate the dispersed phase can be found in Arcen et al. (2004, 2006), therefore, only the main characteristics will be presented hereafter. In the present study, the aerodynamic forces considered are the non-linear drag and the shear-induced lift force, both of them are corrected for near-wall effects Arcen et al. (2006). It has to be noted that the numerical predictions of the particle-laden channel flow by the present code have been evaluated, in the frame of an international test case, against the DNS data issuing from the computational codes of the following participant groups: C. Marchioli and A. Soldati; J.G.M. Kuerten; G. Goldensohn and K. Squires; M. Cargnelutti and L.M. Portela (for further details see Marchioli et al., 2007). The comparison of the results obtained by these DNS codes, which are based on different numerical methods, has shown the accuracy of the present code. The simulations were run for three sets of particles characterized by different Stokes particle response times in wall units,<sup>1</sup>  $\tau_p^+ = 2, 15.4$  and  $27.1$ . The corresponding dimensionless diameters are  $d_p/\delta = 0.5 \times 10^{-3}, 1.4 \times 10^{-3}$  and  $1.4 \times 10^{-3}$ , for which density ratios,  $\rho_p/\rho_f$ , are 4166, 4166 and 7333 respectively. For the initialization and the computation of the statistics of the dispersed phase, the domain is divided in the wall-normal direction into 128 slices, with the slice thickness being equal to the wall-normal grid spacing. Initially, 5000 solid particles were distributed randomly in each slice,

and their initial velocity was set equal to the surrounding fluid velocity. Therefore, the total number of particles was thus 640,000 and the concentration is not initially uniform. Statistics on the dispersed phase were started after a time lag of approximately  $t^+ = 600$  to get results independent of the imposed initial conditions.

It has to be noted that the DNS data used in the present study have been extracted while particle statistics had not already reached a stationary state since it requires a large amount of time. Nevertheless, we think that the particle statistics (with the exception of the mean concentration) obtained in such a state are not significantly different from those we have extracted after a time lag of approximately  $t^+ = 600$ . Moreover, we have taken the non-linear drag force corrected for near-wall effects and the optimum lift force into account in the equation governing the motion of a particle, whereas the ATS model was derived under the assumption that the Stokes drag law is only of importance. However, it has been shown by Arcen et al. (2006) that, in the absence of external forces, the drag corrections and the use of the optimum lift force have a negligible impact on the dispersed phase statistics such as mean streamwise fluid and particles velocities, root mean square of the particle velocities, fluid Reynolds stresses at particle location and fluid–particle covariance tensor. Therefore, we can presume that the difference in the force considered for the present numerical simulation and for the derivation of the ATS model will not be responsible of significant discrepancies between the DNS and ATS model results.

### 4. Results and discussion

In this section, the results predicted by the modified and original ATS models will be compared to those issuing from DNS and also from one the simplest existing model for particle kinetic stresses and fluid–particle covariances which was derived by Tchen (1947) and Hinze (1975). In the present study, we have considered the extensions of the original model, proposed by Simonin et al. (1993) and Desjonqueres et al. (1986), which enable to link the diagonal components of the particle kinetic stress and fluid–particle covariance tensors to the Reynolds stresses of the fluid seen. It should be noted that the Tchen–Hinze model was established considering the motion of small particles at low particle Reynolds number in stationary homogeneous isotropic turbulence. Consequently, it is expected that this model will not be able to predict correctly all the components of the particle kinetic stress and fluid–particle covariance tensors since only the inertia filtering effect is taken into account in the Tchen–Hinze theory. Finally, it has to be noted that the Tchen–Hinze model presented here is a modified version of the original model since the statistics of the fluid seen by a particle were assumed identical to those obtained along fluid-element trajectories in the studies conducted by Tchen (1947) and Hinze (1975). As for the ATS model, both versions of this model will be evaluated in the present study.

<sup>1</sup> Quantities in wall units are normalized with the viscous scales (i.e. the wall-shear velocity  $u_\tau$  and the viscous lengthscale  $\nu/u_\tau$ ) and will be indicated hereafter by the superscript  $(\cdot)^+$ .

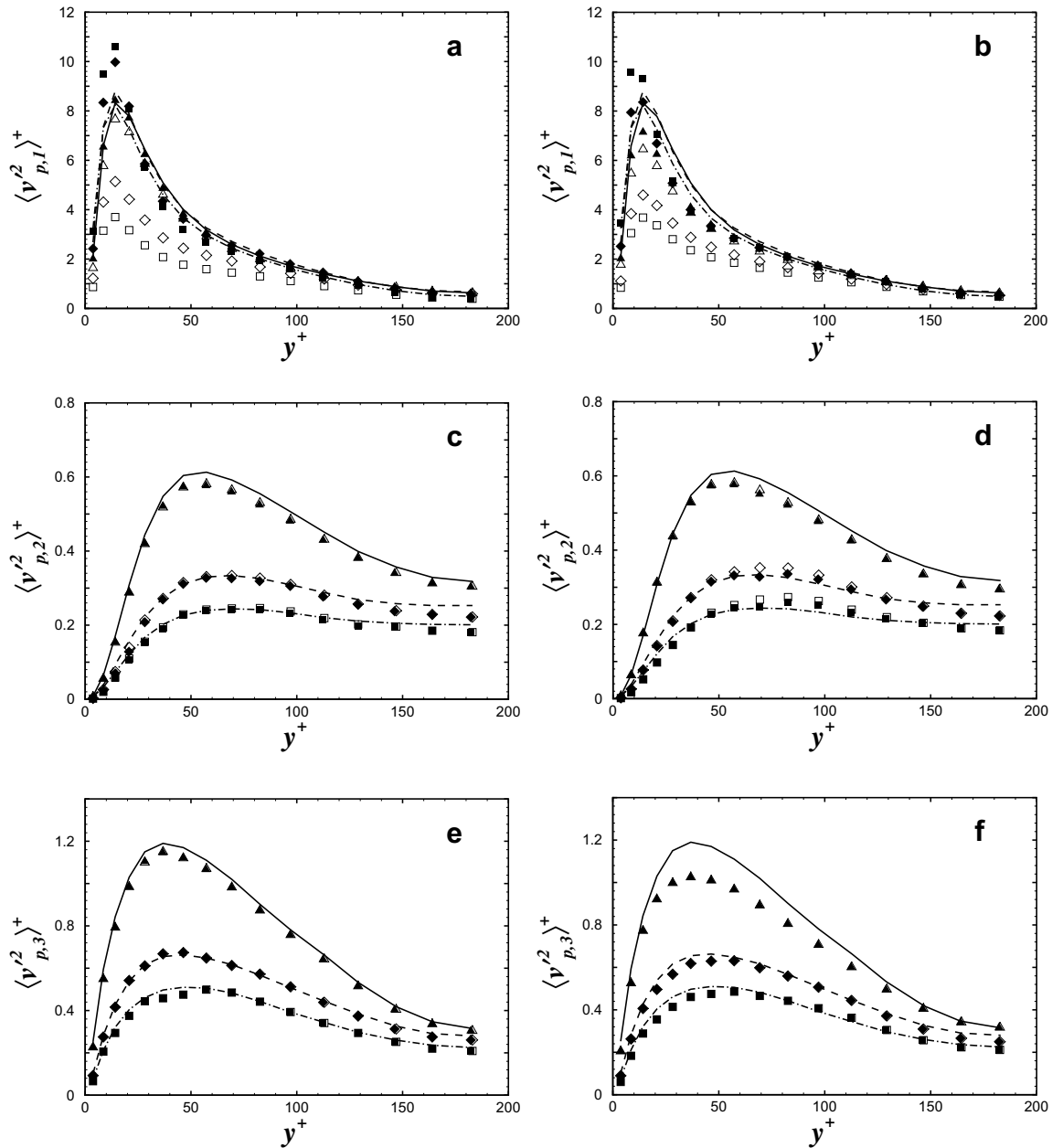


Fig. 1. Prediction of the diagonal components of the particle kinetic stress tensor,  $\langle v_{p,i}^2 \rangle^+$ , by the ATS and Tchen–Hinze models. (a), (c), and (e): using the statistics of the fluid seen by particles. (b), (d), and (f): using the fluid Eulerian and Lagrangian statistics. DNS: —,  $\tau_p^+ = 2$ ; --,  $\tau_p^+ = 15.4$ ; - · -,  $\tau_p^+ = 27.1$ . Tchen–Hinze model:  $\Delta$ ,  $\tau_p^+ = 2$ ;  $\diamond$ ,  $\tau_p^+ = 15.4$ ;  $\square$ ,  $\tau_p^+ = 27.1$ . ATS model: black symbols.

#### 4.1. Particle kinetic stresses

The diagonal components of the particle kinetic stress tensor results issuing from the modified and original ATS and Tchen–Hinze models are plotted as a function of the wall-normal coordinate in Fig. 1.

Concerning the modified ATS and Tchen–Hinze models, it can be observed from Fig. 1a that the modified ATS model overestimates the maximum of the streamwise particle kinetic stress,  $\langle v_{p,1}^2 \rangle^+$ , for  $\tau_p^+ = 15.4$  and 27.1 particles. In contrary to this ATS model, the modified Tchen–Hinze model underestimates the DNS data since it

predicts a decrease of  $\langle v_{p,1}^2 \rangle^+$  as inertia increases, whereas the DNS data show that inertia does not significantly influence this particle kinetic stress. The effect of particle inertia on  $\langle v_{p,1}^2 \rangle^+$ , noted for the DNS data, has been theoretically established in homogeneous shear flows (Liljegren, 1993; Reeks, 1993; Simonin et al., 1995; Zaichik, 1999) and previously reproduced numerically in both homogeneous and non-homogenous shear flows (Simonin et al., 1995; Wang and Squires, 1996; Portela et al., 2002). This phenomenon is mainly due to the production of streamwise particle velocity fluctuation by the mean velocity shear. Therefore,

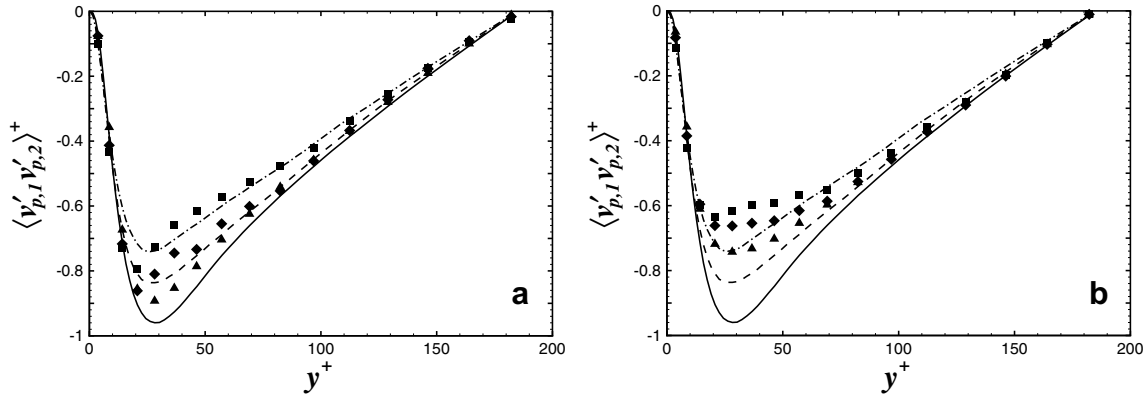


Fig. 2. Prediction of the non-diagonal component of the particle kinetic stress tensor,  $\langle v'_{p,1} v'_{p,2} \rangle^+$ , by the ATS model. (a) Using the statistics of the fluid seen by particles. (b) Using the fluid Eulerian and Lagrangian statistics. DNS: —,  $\tau_p^+ = 2$ ; - - -,  $\tau_p^+ = 15.4$ ; - · - ·,  $\tau_p^+ = 27.1$ . ATS model: same symbols as in Fig. 1.

the Tchen–Hinze model cannot capture it, in contrary to the ATS model in which the production term is present. Nevertheless, despite this crucial effect can be qualitatively reproduced by the ATS model, the values obtained are not in very good accordance with the DNS data. The results obtained for the wall-normal and spanwise particle kinetic stresses, plotted in Fig. 1c and e, shows that the modified ATS and Tchen–Hinze models predict correctly the decrease of these particle stresses with increasing inertia. The results issuing from Tchen–Hinze model for the spanwise component are undistinguishable since the expression used is identical to that of the ATS model. Consequently, we can conclude from the results obtained for  $\langle v_{p,2}^2 \rangle$  and  $\langle v_{p,3}^2 \rangle$  that both models are able to correctly predict the inertia filtering effect.

The comparison between these results and those obtained from the original ATS and Tchen–Hinze models (Fig. 1b, d, and f), i.e. using the fluid Eulerian and Lagrangian statistics, shows that the use of the statistics of the fluid seen by particles does not lead to a significant improvement of the predictions.

The modified ATS model results for the particle kinetic shear stress,  $\langle v'_{p,1} v'_{p,2} \rangle$ , are presented in Fig. 2a. It can be seen that the model results are qualitatively and quantitatively in good accordance with those issuing from the DNS computation. The model is able to correctly predict the decrease of the magnitude of the particle kinetic shear stress as inertia increases.

Concerning the particle kinetic shear stress predictions by the original ATS model, plotted in Fig. 2b, the ATS model predictions are seen to be less satisfactory when the fluid Eulerian and Lagrangian statistics are used since it underpredicts significantly the DNS data for  $10 < y^+ < 100$  whatever the particle inertia.

#### 4.2. Fluid–particle covariances

The performance of the modified and original ATS models to predict the fluid–particle covariance was also

examined. The diagonal components of the fluid–particle covariance tensor,  $\langle \tilde{u}'_i v'_{p,j} \rangle$ , are plotted in Fig. 3 as a function of  $y^+$  for the three different particle sets. The results obtained from the DNS computations and the modified and original Tchen–Hinze models are also reported.

The streamwise fluid–particle covariance,  $\langle \tilde{u}'_1 v'_{p,1} \rangle$ , given by the modified ATS and Tchen–Hinze models are compared to the DNS data in Fig. 3a. It can be noted that both models predict correctly the decrease of the covariance as the particle inertia increases. Nonetheless, a better quantitative agreement is given by the ATS model, since the Tchen–Hinze underpredicts significantly  $\langle \tilde{u}'_1 v'_{p,1} \rangle$  for  $y^+ < 100$  and  $\tau_p^+ = 15.4$  and  $27.1$  particles. The decrease of the streamwise fluid–particle covariance means that the inertia filtering effect is more important than the effect of production of the streamwise velocity fluctuation induced by the mean velocity shear. However, keeping in mind that the Tchen–Hinze model does not take this latter effect into account, it can be concluded from the results given by this model that the production induced by the mean velocity shear cannot be neglected in the fluid–particle covariances models. The wall-normal and spanwise components are plotted in Fig. 3c and e. As expected, the modified ATS and Tchen–Hinze models give similar results which are in good agreement with the DNS data.

The predictions of the original ATS and Tchen–Hinze models are plotted in Fig. 3b, d, and f. It can be seen that the use of the fluid Eulerian and Lagrangian statistics leads to a less accurate estimation of the streamwise component by the ATS and Tchen–Hinze models. The discrepancies are particularly significant for  $\tau_p^+ = 2$  particles. The predictions of the wall-normal and spanwise components are presented in Fig. 3d and f, the results are similar to those obtained from the modified ATS and Tchen–Hinze models. For this latter case, the predictions are noticed to be in better agreement with the DNS data when the statistics of the fluid seen by particles are used in the models.

Finally, the non-diagonal components of the fluid–particle covariance tensor,  $\langle \tilde{u}'_1 v'_{p,2} \rangle$  and  $\langle \tilde{u}'_2 v'_{p,1} \rangle$ , predicted by



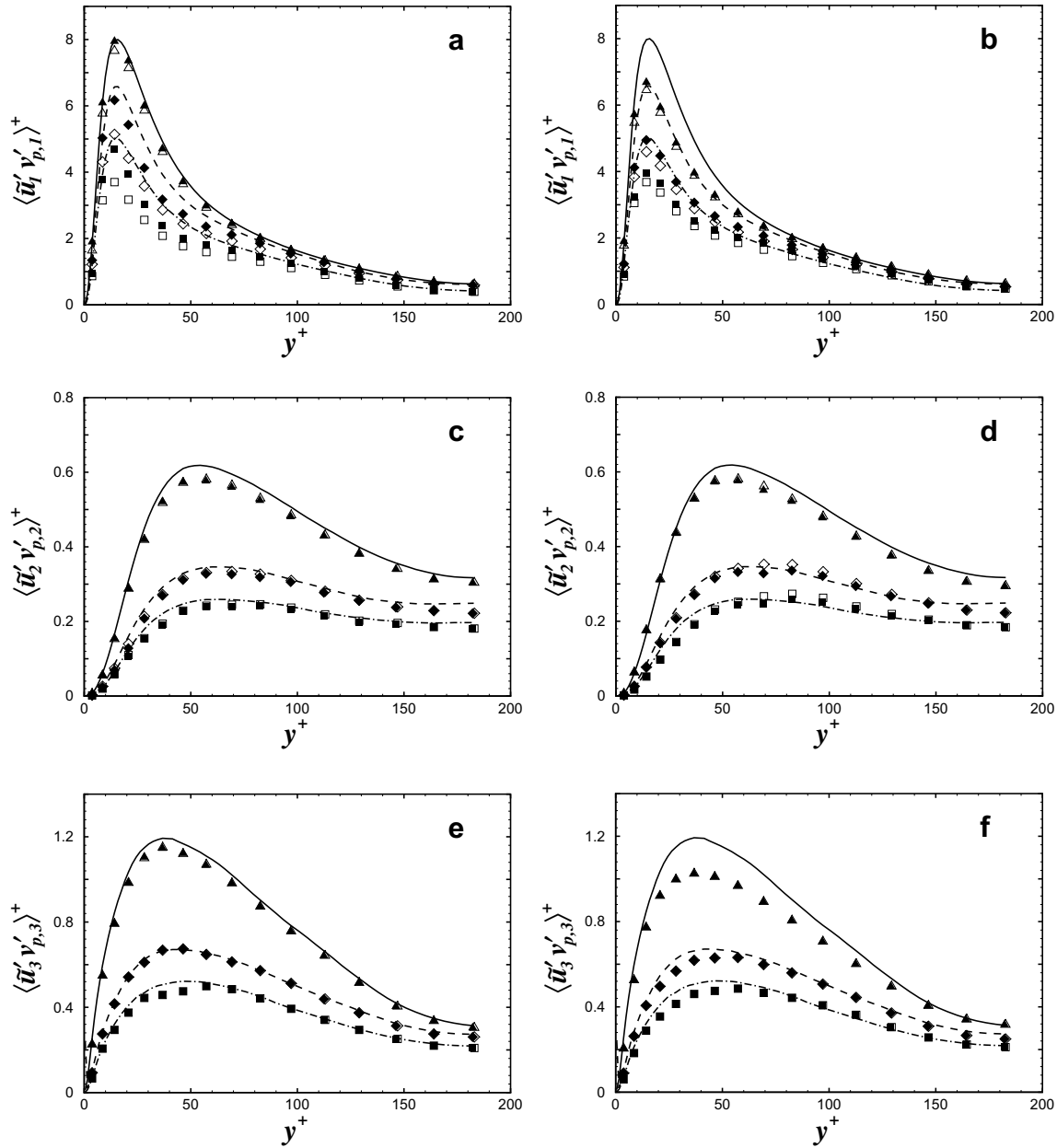


Fig. 3. Prediction of the diagonal components of the fluid–particle covariance tensor,  $\langle \tilde{u}'_i v'_{p,i} \rangle$ , by the ATS and Tchen–Hinze models. (a), (c), and (e): using the statistics of the fluid seen by particles. (b), (d), and (f): using the fluid Eulerian and Lagrangian statistics. DNS: —,  $\tau_p^+ = 2$ ; - - ,  $\tau_p^+ = 15.4$ ; - · - ,  $\tau_p^+ = 27.1$ . Tchen–Hinze model:  $\Delta$ ,  $\tau_p^+ = 2$ ;  $\diamond$ ,  $\tau_p^+ = 15.4$ ;  $\square$ ,  $\tau_p^+ = 27.1$ . ATS model: black symbols.

the modified ATS model are presented in Fig. 4a and c. Note that this tensor is asymmetric. This property has been theoretically established in homogeneous shear flows (Simonin et al., 1995; Zaichik, 1999) and also previously observed from Large Eddy Simulations of both homogeneous and non-homogeneous shear flows (Simonin et al., 1995; Wang and Squires, 1996). From the present DNS results, it is seen that the asymmetry becomes more pronounced as the inertia increases, more precisely, the magnitude of  $\langle \tilde{u}'_1 v'_{p,2} \rangle$  decreases more rapidly than that of  $\langle \tilde{u}'_2 v'_{p,1} \rangle$ . This effect is captured by the ATS model. Nevertheless, the model strongly underpredicts  $\langle \tilde{u}'_1 v'_{p,2} \rangle$  whatever

the particle inertia while the influence of the particle inertia on  $\langle \tilde{u}'_2 v'_{p,1} \rangle$  is seen to be correctly estimated. A possible explanation of the too strong decrease of  $\langle \tilde{u}'_1 v'_{p,2} \rangle$  is that there is no production term by a mean velocity gradient in the expression derived from the ATS model.

The results issuing from the original ATS model are presented in Fig. 4b and d. Similarly to the modified ATS model, the original model, written in terms of the fluid Eulerian and Lagrangian statistics, does not estimate correctly the influence of the particle inertia on  $\langle \tilde{u}'_1 v'_{p,2} \rangle$ . The results obtained for  $\langle \tilde{u}'_2 v'_{p,1} \rangle$  are in acceptable agreement with the

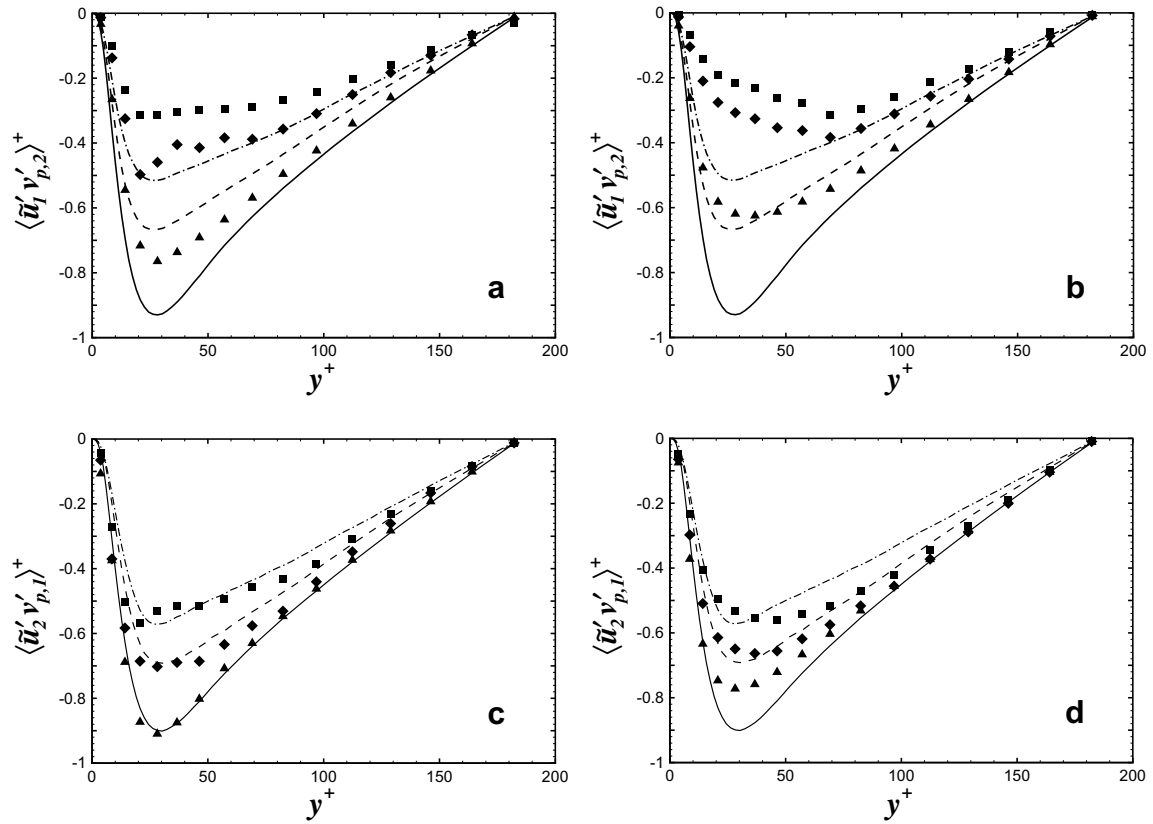


Fig. 4. Prediction of the non-diagonal components of the fluid–particle covariance tensor,  $\langle \tilde{u}'_1 v'_{p,2} \rangle$  and  $\langle \tilde{u}'_2 v'_{p,1} \rangle$ , by the ATS model. (a) and (c): using the statistics of the fluid seen by particles. (b) and (d): using the fluid Eulerian and Lagrangian statistics. DNS: —,  $\tau_p^+ = 2$ ; --,  $\tau_p^+ = 15.4$ ; - · -,  $\tau_p^+ = 27.1$ . ATS model: same symbols as in Fig. 3.

DNS data and the asymmetry of the fluid–particle tensor is still reproduced by the original model. Nevertheless, a more satisfactory prediction was obtained using the statistics of the fluid seen by particles whatever the particle inertia.

## 5. Conclusion

The objective of the present paper was to study the performance of relations for the mean turbulent particle and fluid–particle properties, derived by Zaichik et al. (2004) from a statistical model, in non-homogeneous turbulence. To evaluate the ATS model performance in a non-homogeneous turbulent flow, results obtained from a direct numerical simulation of a gas-particle channel flow are compared to those predicted by the model. In the present study, the performance of two versions of the ATS model have been evaluated to determine the importance of accounting for the statistical bias that exist between the turbulent properties of the fluid viewed by a particle and those of the fluid in the specification of the parameters appearing in the model. In the first one, the inertia and crossing trajectory effects on the turbulent properties of the fluid viewed by a particle is taken into account, whereas the second version is identical to the one originally proposed by Zaichik et al. (2004). The comparison has shown that the modified ATS model predicts correctly the influence of the particle inertia on the

four components of particle kinetic stress tensor, even if the maximum of the streamwise component was found to be overpredicted. In addition, this model has been seen to be able to predict correctly the decrease of the diagonal components of fluid–particle covariance tensor as inertia increases and to reproduce the asymmetry of this tensor. Nonetheless, one of the non-diagonal components has been shown to be largely underestimated. It should be emphasized that a major drawback of the modified version of the ATS model used in the present study is that it necessitates to know beforehand the timescales of the fluid seen when no external force is acting on particles and some mean and turbulent statistics of the fluid viewed by a particle. Unfortunately, no models of these characteristics exist for non-homogeneous turbulence. Further studies of these parameters in such a turbulence are thus needed to improve continuum two-fluid models. Nevertheless, this study has shown that using the original version of the ATS model, in which the inertia effect on the parameters appearing in the model is neglected, leads to a less satisfactory prediction, however, the results are still in acceptable agreement with the DNS data. Consequently, as a first approximation in a non-homogeneous flow, one could overcome the problem of knowing a priori the statistical features of the fluid velocity seen by the particles by considering that the Eulerian statistics of the fluid are identical to

those of the fluid seen. The original ATS model could be thus used in association with a model of the Lagrangian timescales which reflects correctly the directional dependence of these timescales, see Oesterlé and Zaichik (2004) for instance, while the mean velocity and the Reynolds stresses of the carrier flow could be obtained by means of a second-moment closure model.

Finally, as expected, it has been seen that the simple Tchen–Hinze model, which has been developed for stationary homogeneous isotropic turbulence, does not estimate correctly the streamwise components of the particle kinetic stresses and fluid–particle covariances. This points out the importance of taking the production of the streamwise particle velocity fluctuation by the mean velocity shear into account for assessing the particle kinetic stresses and of the fluid–particle covariances in non-homogeneous turbulent flows.

The next step of the present study should be to conduct a similar comparison in the presence of an external force field to examine the importance of taking the crossing trajectory effects on the parameters appearing in the ATS model.

## References

- Arcen, B., Tanière, A., Oesterlé, B., 2004. Numerical investigation of the directional dependence of integral time scales in gas–solid channel flow. In: Proceedings of the 5th International Conference on Multiphase Flow. Paper no. 297. Yokohama, Japan.
- Arcen, B., Tanière, A., Oesterlé, B., 2006. On the influence of near-wall forces in particle-laden channel flows. *Int. J. Multiphase Flow* 32, 1326–1339.
- Desjonqueres, P., Gouesbet, G., Berlemont, A., Picart, A., 1986. Dispersion of discrete particles by continuous turbulent motions: new results and discussions. *Phys. Fluids* 29, 2147–2151.
- Ferry, J., Rani, S.L., Balachandar, S., 2003. A locally implicit improvement of the equilibrium Eulerian method. *Int. J. Multiphase Flow* 29, 869–891.
- Hinze, J.O., 1975. *Turbulence*, second ed. McGraw-Hill, New-York.
- Liljegren, L.M., 1993. The effect of a mean fluid velocity gradient on the streamwise velocity variance of a particle suspended in a turbulent flow. *Int. J. Multiphase Flow* 19, 471–484.
- Marchioli, C., Soldati, A., Kuerten, J.G.M., Arcen, B., Tanière, A., Goldensohn, G., Squires, K.D., Cargnelutti, M.C., Portela, L.M., 2007. Statistics of particle dispersion in direct numerical simulations of wall-bounded turbulence: results of an international collaborative benchmark test. In: Proceedings of the 6th International Conference on Multiphase Flow. ICMF, Leipzig, Germany.
- Oesterlé, B., Zaichik, L.I., 2004. On Lagrangian time scales and particle dispersion modeling in equilibrium shear flows. *Phys. Fluids* 16, 3374–3384.
- Orlandi, P., 2000. *Fluid Flow Phenomena. A Numerical Toolkit*. Kluwer Academic Publishers.
- Portela, L.M., Cota, P., Oliemans, R.V.A., 2002. Numerical study of the near-wall behavior of particles in turbulent pipe flows. *Powder Tech.* 125, 149–157.
- Reeks, M.W., 1993. On the constitutive relations for dispersed particles in nonuniform flows. I: dispersion in a simple shear flow. *Phys. Fluids* 5, 750–761.
- Simonin, O., Deutsch, E., Boivin, M., 1995. Large eddy simulation and second-moment closure model of particle fluctuating motion in two-phase turbulent shear flows. In: Durst, F. et al. (Eds.), *Selected Papers from the 9th International Symposium on Turbulent Shear Flows*. Springer-Verlag, Berlin, p. 85.
- Simonin, O., Deutsch, E., Minier, J.P., 1993. Eulerian prediction of fluid–particle correlated motion in turbulent two-phase flows. *Appl. Scientific Res.* 51, 275–283.
- Tchen, C.M., 1947. Mean value and correlation problems connected with the motion of small particles suspended in a turbulent fluid. Ph.D. Thesis, University of Delft, The Hague.
- Wang, L., Stock, D.E., 1993. Dispersion of heavy particles by turbulent motion. *J. Atmos. Sci.* 50, 1897–1913.
- Wang, Q., Squires, K.D., 1996. Large eddy simulation of particle-laden turbulent channel flows. *Phys. Fluids* 8, 1207–1223.
- Zaichik, L.I., 1999. A statistical model of particle transport and heat transfer in turbulent shear flows. *Phys. Fluids* 11, 1521–1534.
- Zaichik, L.I., Oesterlé, B., Alipchenkov, V.M., 2004. On the probability density function model for the transport of particles in anisotropic turbulent flow. *Phys. Fluids* 16, 1956–1964.